IMPACT OF FADING ON THE PERFORMANCE OF ALOHA AND CSMA

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ABSTRACT

This paper considers the performance of the ALOHA and CSMA MAC protocols in wireless ad hoc networks in the presence of fading. Increasing the rate of successful reception of packets is our objective, and thus, outage probability is used as the performance evaluation metric. In our network model, packets belonging to specific transmitters arrive randomly in space and time according to a 3-D Poisson point process, and are then transmitted to their intended destinations using a fully-distributed MAC protocol. A packet transmission is considered successful if the received SINR is above a predefined threshold for the duration of the packet. Approximate expressions are derived for the outage probability of ALOHA and the different flavors of CSMA, namely CSMA with transmitter sensing, receiver sensing, and joint transmitter-receiver sensing. The introduction of fading adds to the hidden and exposed node problems of CSMA, resulting in an up to 75% increase in the outage probability. Interestingly, however, the relative difference between the protocols remains unchanged.

1. INTRODUCTION

One of the main challenges in the design of wireless networks is the existence of interference between communication links. This challenge is intensified in wireless *ad hoc* networks, as the the number and the exact positions of the interferers are unknown. This is because ad hoc networks are self-configuring and can rapidly be formed and deployed from whatever nodes available. However, in many cases, we have information on the distribution of the interferers. In dense networks with indiscriminate node locations, a common and analytically convenient assumption is that the node distribution follows a homogeneous Poisson point process (PPP) with a known intensity, and this is what we assume in our model.

The issue of interference is more involved than simply estimating the location of interferers. In most networks, there is also the phenomenon of *fading*, i.e., rapid variations of received signal power due to constructive and destructive addition of the signal's multipath components. The introduction of fading often results in degradation of the average performance of networks and complicates the estimation of the interference powers. In order to reduce the interference in a communication system, and thereby increase the probability of correct reception of packets, medium access control (MAC) protocols are often applied. Two of the most popular MAC protocols are ALOHA and Carrier-Sensing Multiple Access (CSMA), on which we focus in this paper.

The ad hoc network model we consider entails a Poisson distribution of transmitter (TX) locations in space and of packet arrivals in time. Each TX communicates with a receiver (RX) a certain distance away from it, and transmits its packets according to the specified MAC protocol. As in [1]– [3], we focus exclusively on single-hop communication. All multiuser interference is treated as noise, and the communication links exhibit Rayleigh fading. We are interested in correct reception of packets, and we use SINR to evaluate the performance of our network. Accounting for the sources of randomness in the model (location of nodes, concurrent transmissions, and fading), we investigate the relationship between outage probability (OP), transmission density, and the choice of MAC protocol. We wish to establish an understanding for the influence of fading on the performance of MAC protocols in wireless ad hoc networks.

1.1. Related Work

Numerous works have been done on fading and its effects in communication networks. In [4], Baccelli et. al. evaluate the outage capacity of ALOHA in a Rayleigh-fading environment. The focus of their work is on optimizing the transmit power and the access probability (which is the product of the number of simultaneously successful transmissions per unit space by the average range of each transmission). In [5], the success probability ($P_s = 1 - \text{OP}$) of slotted ALOHA is derived in various fading scenarios, obtaining similar results as the formulas we apply in our analysis. In [2], a new fading model is proposed that combines uncertainties in the transmission distance with small-scale fading. In this model, where TXs are Poisson distributed and fading is assumed to be Rayleigh, it is proven that the effect of fading is a *thinning* in the geographial domain.

Ilow and Hatzinakos [6] consider the impact of random channel effects on the aggregate cochannel interference in an ad hoc network where nodes are distributed according to a PPP. Whereas their focus is on identifying the impact of fading on the parameters of the characteristic function of the interference, we concentrate on the MAC layer design and evaluate the OP of various MAC protocols. In [7] and [8], the transmission capacity of slotted ALOHA in an ad hoc network with Poisson distributed node locations is evaluated for a given outage requirement. It is shown that in the absence of CSI, fading can significantly reduce the transmission capacity [8]. Rather than setting a constraint on the OP, we evaluate the performance of our ad hoc network in terms of OP. This evaluation is performed for both the ALOHA and CSMA protocols.

For our analysis, we apply some of the techniques that were proposed in [9] and developed further in [1] and [10]. Due to the absence of fading in these works, outage could be directly translated into a distance problem. That is, lower bounds to the OP were derived by only considering the *closest* interferer to the node under observation. In the presence of fading, however, as is the case in this work, the closest interferer does not necessarily cause outage for our packet. It is now the *dominant* interferer (i.e., the one with the largest received interference power when its link is affected by fading) that should be considered.

2. SYSTEM MODEL

Our model considers an ad hoc network where packets are distributed in space and time according to an inhomogeneous 3-D PPP with intensity $\lambda^s\lambda^t$ [packets/s/m^2], as proposed in [1] and elaborated in [10]. More specifically, packets arrive in time according to a 1-D PPP with intensity $\lambda^t\lambda^s A$ [packets/s], where A is the area of the network. Each packet has a fixed duration T. Upon the arrival of a packet, it is assigned to a TX, which is then randomly placed on a 2-D plane, yielding a homogeneous PPP with density $\lambda = \lambda^s\lambda^t T$ [packets/m^2]. According to the specified MAC protocol, the packet is then transmitted with a constant power ρ to its intended RX, which is assumed to lie a *fixed* distance R away. The fixed distance between TX and RX is clearly not a natural assumption. However, the whole network with a fixed R could be viewed as a snapshot of a multi-hop wireless network, where R is the bounded average inter-relay distance designed by the routing protocol. When the packet has been served (successful or not), the corresponding TX-RX pair disappears from the plane. Such 3-D representation of the network simplifies our model as it allows us to consider a *single* random process describing both the temporal and spatial variations of the system.

For the channel model, we consider path loss attenuation effects (with exponent $\alpha > 2$) in addition to fading effects, modelled as Rayleigh fading. Each node potentially sees interference from all TXs on the plane, and these independent interference powers are added to the channel noise power, η, resulting in:

$$\text{SINR} = \frac{\rho h_{00} R^{-\alpha}}{\eta + \sum_i \rho h_{0i} r_i^{-\alpha}}, \tag{1}$$

where r_i and h_{0i} are, respectively, the distance and fading coefficient between the node under observation (this could be either the TX or RX of the packet we are considering) and the i-th interfering TX. h_{00} represents the fading effects between the RX under observation and its designated TX. The summation is over all active interfering transmissions at a particular snapshot in time. If this measured SINR falls below the required threshold β at any time during the packet transmission, the packet is received in outage.

The transmissions occur according to either ALOHA or CSMA. In the ALOHA protocol, packets are simply transmitted to their destinations regardless of the channel condition. In the CSMA protocol, the channel is sensed at the beginning of the packet, and if the measured SINR is above β, the packet is transmitted. Otherwise, it is dropped. No retransmissions are applied in our model.

3. OUTAGE PROBABILITY OF ALOHA

The ALOHA protocol was proposed by Norman Abramson in 1970, and is one of the simplest MAC algorithms used for communication. This protocol allows packets to be transmitted to their intended RXs regardless of channel conditions and the probability of collision. Since our model does not allow for retransmissions, a packet received erroneously (i.e., if the SINR at the RX falls below the threshold β any time during the packet duration) on its first try is immediately counted as a source of outage. Clearly, due to the high level of interference in the network, this algorithm results in a high OP. To improve the throughput of ALOHA, *slotted ALOHA* was developed. In this protocol, a packet can only be transmitted at the start of the next time slot after it has been formed. This reduces the OP as there is no longer partial overlap of packets. This improvement is, however, at the expense of a need for synchronization. Not only is this costly, but having a slotted system also introduces additional delays in the reception of packets.

In [7], the lower bound to the OP of slotted ALOHA in a non-faded network was derived to be:

$$P_{out}^{LB}(\text{Slotted ALOHA}) = 1 - e^{-\pi\lambda s^2}, \tag{2}$$

where $s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{1}{\alpha}}$. Note that the exponent $(\pi\lambda s^2)$ is in fact the expected number of active interferers inside the guard zone [9] of the RX under observation, RX_0.

By the introduction of fading, we can no longer use the concept of guard zones and distance to evaluate the OP. Instead, we consider the probability of having a *dominant* interferer, i.e., one whose received interference power is strong enough to cause outage for RX_0. This yields a lower bound to the OP, and is derived in [3] to be:

$$\begin{aligned} P_{out}^{f}(\text{Slotted ALOHA}) &\geq 1 - \mathbb{E}\left[e^{-\pi\lambda h_{0i}^{\frac{2}{\alpha}}\left(\frac{h_{00}R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{2}{\alpha}}}\right] \\ &\approx 1 - \exp\left\{-\pi\lambda \mathbb{E}\left[h_{0i}^{\frac{2}{\alpha}}\right] \mathbb{E}\left[\left(\frac{h_{00}R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{2}{\alpha}}\right]\right\}. \end{aligned}$$

As path loss and fading are assumed to be the main sources of signal degradation, we set the noise η to 0. Furthermore, applying the result $\mathbb{E}\left[h_{00}^{-2/\alpha}\right]\mathbb{E}\left[h_{0i}^{2/\alpha}\right] = \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}$, yields [7]:

$$P_{out}^{f}(\text{Slotted ALOHA}) \approx 1 - e^{-\pi\lambda R^2 \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}}. \tag{3}$$

The work of [7] only considers a slotted system. In the following, we extend the techniques and results of [7] to derive the OP of continuous-time protocols. To find an expression for the OP of *unslotted* ALOHA, we note that outage occurs if the received SINR falls below β at any time during its transmission. This means that rather than considering packet arrivals only in $[-T, 0]$, we now consider the period $[-T, T]$. Based on similar reasoning as in [1], and by applying Eq. (3), we derive the OP to be:

$$P_{out}^{f}(\text{Unslotted ALOHA}) \approx 1 - e^{-2\pi\lambda R^2 \frac{2\pi/\alpha}{\sin(2\pi/\alpha)}}. \tag{4}$$

Compared to Eq. (3), the OP of unslotted ALOHA is for lower densities a factor of 2 larger than that of slotted ALOHA.

4. OUTAGE PROBABILITY OF CSMA

By introducing sensing of the channel prior to transmission, the CSMA protocol improves the performance of ALOHA considerably. In this protocol, when a new packet is formed, the channel is sensed (by the TX in CSMA_{TX}, by the RX in CSMA_{RX}, and by both TX and RX in $\text{CSMA}_{\text{TXRX}}$), and a decision is made on whether or not the packet transmission should be initiated. If the measured SINR at the start of the packet is above β, then the packet is sent immediately. Otherwise, the transmission is backed off. Since no retransmissions are allowed, a backoff is equivalent to dropping the packet. Consequently, each backoff must necessarily be counted as outage. In the following, we derive approximate expressions for the OP of the different flavors of CSMA mentioned above.

4.1. CSMA with RX Sensing

In this version of the CSMA protocol, the backoff decision is made by the RX. If the measured SINR at the RX is less than β at the start of the packet, it backs off from transmission. The communication between each TX and its RX is assumed to occur over a separate control channel, meaning that there is no interference between these signals and the data packets[1]. Moreover, we assume that the delay caused from this exchange of information is minimal. In [1], it was shown that the simple feedback channel in $\mathrm{CSMA}_{\mathrm{RX}}$ results in up to 23% less OP compared to the other protocols.

The total OP of $\mathrm{CSMA}_{\mathrm{RX}}$ is given by:

$$\begin{aligned} &P_{out}^f(\mathrm{CSMA}_{\mathrm{RX}}) \\ &= \Pr\left[\mathrm{SINR}_0 < \beta \text{ at } t=0 \cup \mathrm{SINR}_0 < \beta \text{ at some } t \in (0,T]\right] \\ &= P_b^f + (1-P_b^f)\, P_{out}^f(\mathrm{CSMA}_{\mathrm{RX}}|\text{active}), \end{aligned} \tag{5}$$

To derive the backoff probability of $\mathrm{CSMA}_{\mathrm{RX}}$, P_b^f, we note that the sensing at the start of the packet is dependent on the number of packet arrivals during the last T seconds. Thus, we apply Eq. (3), with the difference that the density of active packets on the plane is now a function of the backoff probability, namely $\lambda_{csma} \approx \lambda_{aloha}(1-\tilde{P}_b^f)$. We assume that the active interferers are still Poisson distributed, an approximation that is proven to be reasonable by the simulation results. Hence:

$$\tilde{P}_b^f = 1 - \exp\left\{-\pi\lambda R^2\left(1-\tilde{P}_b^f\right)\frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right\}. \tag{6}$$

Solving for $\tilde{P}_b^f$ in terms of the Lambert function, $W_0(\cdot)$, yields:

$$\tilde{P}_b^f = 1 - \frac{W_o\left(\lambda\pi R^2\frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right)}{\lambda\pi R^2\frac{2\pi/\alpha}{\sin(2\pi/\alpha)}}. \tag{7}$$

For low densities, the backoff probability of CSMA is small, and so $1-P_b^f \approx 1$. Eq. (6) then becomes equal to Eq. (4).

Next, we derive an expression for the probability that a packet goes into outage at some time during its transmission, denoted by $P_{out}^f(\mathrm{CSMA}_{\mathrm{RX}} \mid \text{active})$. This is found by considering all the active dominant interferers, TX_i, on the plane (i.e., all TXs that started their transmission during the last T seconds, and that can alone cause outage for the ongoing packet transmission of TX_0-RX_0):

$$\begin{aligned} \mu &= \text{Intensity of active dominant interferers for } \mathrm{RX}_0 \\ &= \Pr\left(\mathrm{TX}_i \text{ placed at } (x,y) \text{ during } (0,T]\right) \\ &\quad \cdot \Pr(\mathrm{TX}_i \text{ activated}|(x,y)) \\ &\quad \cdot \Pr(\mathrm{TX}_i \text{ causes outage for } \mathrm{RX}_0|\mathrm{TX}_i \text{ active at } (x,y)). \end{aligned} \tag{8}$$

The first term in (8) is equal to $\lambda = \lambda^s\lambda^t T$ [1]. To derive the second term, we assume that the interferer makes its backoff decision based on the interference from TX_0 only. This approximation is reasonable, because once TX_i is a dominant interferer for RX_0, it is likely that TX_0 is the dominant interferer for RX_i (due to the distance-dependence of OP). Moreover, we assume that the distance TX_i-RX_0 is approximately equal to the distance TX_0-RX_i, denoted by r. This yields:

$$\begin{aligned} \Pr(\mathrm{TX}_i \text{ activated}|(x,y),h_{ii}) &= \Pr(\mathrm{SINR}_i \geq \beta|(x,y),h_{ii}) \\ &\geq \Pr(\mathrm{SINR}_i \geq \beta \text{ based on } \mathrm{TX}_0 \text{ only}|(x,y),h_{ii}) \\ &= \Pr(h_{i0} < \frac{h_{ii}R^{-\alpha}r^{\alpha}}{\beta}|(r,\phi),h_{ii}) \\ &= F_{H_i}\left(\frac{h_{ii}R^{-\alpha}r^{\alpha}}{\beta}\right), \end{aligned} \tag{9}$$

[1]The information on whether or not to transmit is binary, meaning that the control channel will consume an insignificant portion of the bandwidth.

where $F_{H_i}(\cdot)$ is the CDF of H_i. Finally, we derive an expression for the last term of Eq. (8):

$$\begin{aligned} &\Pr(\mathrm{TX}_i \text{ causes outage for } \mathrm{RX}_0 \mid \mathrm{TX}_i \text{ active at } (x,y), h_{00}) \\ &\quad= \Pr(h_{0i} > \frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta} \mid \mathrm{TX}_i \text{ active at } (r,\phi), h_{00}) \\ &\quad= 1 - F_{H_i}\left(\frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta}\right). \end{aligned} \tag{10}$$

Inserting Eqs. (9) and (10) back into Eq. (8) yields:

$$\mu \approx \lambda\, F_{H_i}\left(\frac{h_{ii}R^{-\alpha}r^{\alpha}}{\beta}\right)\left[1 - F_{H_i}\left(\frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta}\right)\right].$$

The expected total number of interferers that can alone cause outage for the packet of RX_0 is then:

$$\begin{aligned} &\mathbb{E}[\#\text{ dominant interferers for } \mathrm{RX}_0 \mid h_{00}, h_{ii}] = \iint_A \mu r dr d\phi \\ &= \int_0^{\infty}\int_0^{2\pi} \lambda F_{H_i}\left(\frac{h_{ii}R^{-\alpha}r^{\alpha}}{\beta}\right)\left[1 - F_{H_i}\left(\frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta}\right)\right] r d\phi dr. \end{aligned}$$

Assuming we have a Rayleigh fading channel, H_i is exponentially distributed, i.e., $f_{H_i}(h_i) = e^{-h_i}$. Based on the Poisson distribution of packets, we derive the approximate probability that a packet goes into outage some time during its transmission to be:

$$\begin{aligned} \tilde{P}_{out}^f(\mathrm{CSMA}_{\mathrm{RX}}|\text{active}) &= 1 - \mathbb{E}_{h_{00},h_{ii}}\left[e^{-\iint_A \mu r dr d\phi}\right] \quad (11) \\ &\approx 1 - \mathbb{E}_{h_{00}}\left[e^{-\int_0^{\infty} 2\pi\lambda\left(1-e^{-\frac{\mathbb{E}[h_{ii}]R^{-\alpha}r^{\alpha}}{\beta}}\right)e^{-\frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta}} r dr}\right]. \end{aligned}$$

The total OP of $\mathrm{CSMA}_{\mathrm{RX}}$ is then found by inserting Eqs. (7) and (11) into Eq. (5).

4.2. CSMA with TX Sensing

This is the conventional CSMA protocol, in which the TX senses its channel and determines whether or not to initiate its transmission. In this protocol, a packet is counted to be in outage when the measured SINR falls below β at the start of the packet either at the TX (in which case, it backs off) or at the RX, and when the SINR at the RX falls below β at some $t \in (0,T]$. As derived in [1], this is:

$$\begin{aligned} P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}) &= P_b^f + (1-P_b^f)P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}|\text{active}) \\ &\quad + P_b^f\left[1 - P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}|\text{active})\right] \\ &\quad - \left[1 - P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}|\text{active})\right]\Pr(\text{RX beg.} \cap \text{TX beg.}), \end{aligned} \tag{12}$$

where P_b^f is the backoff probability, $P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}|\text{active})$ is the probability that the packet goes into outage at some $t \in (0,T]$, and $\Pr(\text{RX beg.} \cap \text{TX beg.})$ is the probability that both the TX and RX are in outage upon arrival.

The backoff probability of $\mathrm{CSMA}_{\mathrm{TX}}$ is equal to that of $\mathrm{CSMA}_{\mathrm{RX}}$, because whether we choose to look at the TX or the RX as the decision-maker, they are both randomly placed on the plane, and their interferers follow the same PPP. This is given by Eq. (6). Moreover, since the backoff probability is equal in $\mathrm{CSMA}_{\mathrm{TX}}$ and $\mathrm{CSMA}_{\mathrm{RX}}$, so is the number of packets that arrive during $(0,T]$ and cause outage for the active transmission of RX_0. Consequently:

$$P_{out}^f(\mathrm{CSMA}_{\mathrm{TX}}|\text{active}) = P_{out}^f(\mathrm{CSMA}_{\mathrm{RX}}|\text{active}), \tag{13}$$

which is given by Eq. (11). Finally, we derive an expression for $P_{out}^{f}(\text{RX beg.} \cap \text{TX beg.})$, the probability that the TX and RX are both in outage upon packet arrival, through manipulation of terms:

$$\begin{aligned}\Pr(\text{RX beg.}) = \Pr(\text{TX beg.}) \Pr(\text{RX beg.}|\text{TX beg.}) \\ + \Pr(\overline{\text{TX beg.}}) \Pr(\text{RX beg.} \mid \overline{\text{TX beg.}}).\end{aligned} \tag{14}$$

We have that $\Pr(\text{RX beg.}) \approx \tilde{P}_b^f$ and $\Pr(\overline{\text{TX beg.}}) \approx 1 - \tilde{P}_b^f$. $\Pr(\text{RX beg.} \mid \overline{\text{TX beg.}})$ is the probability that RX_0 is in outage at the start of the packet, given that TX_0 is not (i.e., given that the transmission is activated). To find this, we consider all activated packet arrivals during the past T seconds that would cause outage for RX_0 at time 0. This is $\iint_A \mu r dr d\phi$, where μ is given by (8). Due to the Poisson distribution of interferers, we have:

$$\Pr(\text{RX beg.}|\overline{\text{TX beg.}}) \approx 1 - \exp\left\{-\iint_A \mu r dr d\phi\right\}.$$

Note that this is indeed the same expression as Eq. (11). Following the discussions above and rearranging (14) results in:

$$\Pr(\text{RX beg.} \cap \text{TX beg.}) \approx \tilde{P}_b^f - (1-\tilde{P}_b^f)\tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}|\text{active}).$$

Inserting this into Eq. (12) yields an approximate expression for the total OP of CSMA_{TX}:

$$\begin{aligned}\tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}) = \tilde{P}_b^f + \tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}|\text{active}) \\ \cdot \left[1 - \tilde{P}_b^f\right]\left[2 - \tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}|\text{active})\right],\end{aligned} \tag{15}$$

where $\tilde{P}_b^f$ and $\tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}|\text{active})$ are given by Eqs. (7) and (11), respectively.

Note that when the TX makes the backoff decision, it might overlook the presence of some interferers that can cause outage for its RX. This adds to the hidden node problem of CSMA, as compared to CSMA_{RX}. Furthermore, the TX might back off in situations where its RX will in fact not be in outage. This adds to the exposed node problem. Hence, CSMA_{TX} is expected to yield a higher OP than CSMA_{RX}, as our simulation results confirm in Section 5.

4.3. CSMA with TX and RX Sensing

In the $\text{CSMA}_{\text{TXRX}}$ protocol, both the TX and RX sense the channel and decide collectively whether or not to back off. Despite the intuition that this protocol must perform better than the versions discussed above, we will see that it actually yields a higher OP compared to CSMA_{RX}. This is due to the exposed node problem introduced by the TX sensing.

With similar reasoning as for CSMA_{RX}, the approximate total OP of $\text{CSMA}_{\text{TXRX}}$ is given by:

$$\begin{aligned}P_{out}^f(\text{CSMA}_{\text{TXRX}}) \\ \approx \tilde{P}_{b2}^f + (1-\tilde{P}_{b2}^f)\tilde{P}_{out}^f(\text{CSMA}_{\text{TXRX}}|\text{active}),\end{aligned} \tag{16}$$

where $\tilde{P}_{b2}^f$ is the backoff probability of $\text{CSMA}_{\text{TXRX}}$. Backoff occurs when the measured SINR at the start of the packet is below β at either the TX, the RX, or both. That is:

$$\begin{aligned}P_{b2}^f = \Pr(\text{TX beg.}) + \Pr(\text{RX beg.}) - \Pr(\text{TX beg.} \cap \text{RX beg.}) \\ \approx 2\,\tilde{P}_b^f - \Pr(\text{RX beg.} \cap \text{TX beg.}),\end{aligned} \tag{17}$$

where $\tilde{P}_b^f$ is given by Eq. (6), and $\Pr(\text{RX beg.} \cap \text{TX beg.})$ was derived in Section 4.2. Hence:

$$\tilde{P}_{b2}^f = \tilde{P}_b^f + (1-\tilde{P}_b^f)\,\tilde{P}_{out}^f(\text{CSMA}_{\text{TX}}|\text{active}). \tag{18}$$

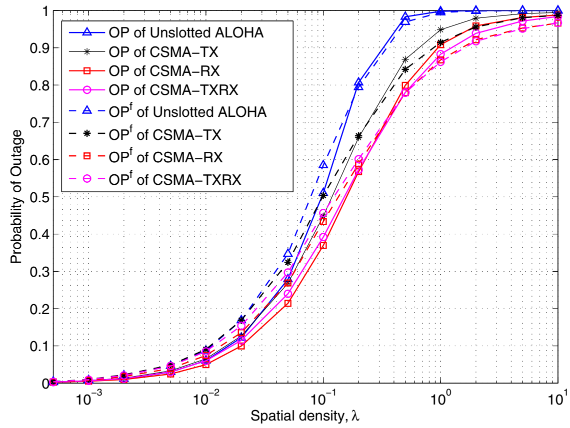

Fig. 1. OP of slotted and unslotted ALOHA, and the different versions of CSMA, as a function of density λ.

The probability that an active transmission is received in outage is derived as in Section 4.1, with the difference that the probability that a dominant interferer, TX_i, activates its transmission is now dependent on the SINR at both TX_i and RX_i. We assume here that the fading coefficient of link TX_0-RX_i is equal to that of link TX_0-TX_i, denoted by h_{i0}. This yields:

$$\begin{aligned}\tilde{P}_{out}^f(\text{CSMA}_{\text{TXRX}}|\text{active}) \\ = 1 - \mathbb{E}_{h_{00}}\left[e^{-\int_0^\infty 2\pi\lambda\left(1-e^{-\frac{\mathbb{E}[h_{ii}]R^{-\alpha}r^{\alpha}}{\beta}}\right)^2 e^{-\frac{h_{00}R^{-\alpha}r^{\alpha}}{\beta}} rdr}\right].\end{aligned} \tag{19}$$

Inserting Eqs. (18) and (19) into Eq. (16) yields the approximate total OP of $\text{CSMA}_{\text{TXRX}}$.

5. NUMERICAL RESULTS

For the simulations, we set R and ρ to be 1, and the path loss exponent $\alpha = 4$. Fig. 1 shows the OP of all the MAC protocols in the presence of fading. As expected, CSMA_{RX} yields the lowest OP of all the continuous-time protocols. For low transmission densities (i.e., when $\lambda \ll 1$), approximately 60% of the total OP of CSMA_{RX} is due to backoff, and 40% is due to outage occurring during transmission. Equivalently, for CSMA_{TX}, these ratios are 50% and 30%, respectively. The effect of fading on CSMA is two-fold: a) It has a direct impact on the backoff probability, and b) once a transmission is activated, it adds to the hidden and exposed node problems. The first can simply be observed by the effect that fading has on ALOHA for low densities. The hidden and exposed node problems are primarily due to the fact that new packet arrivals base their decisions on their own channel conditions, and are not concerned with correct reception of the active packet trasmissions on the plane.

As can be seen from Fig. 2, the conventional CSMA protocol, CSMA_{TX}, does not provide any improvement over ALOHA at low densities. In fact, due to the exposed node problem of CSMA, the OP is 10% higher than that of ALOHA (because the TX backs off in cases where its transmission would not have caused outage for other existing transmissions). As the density of nodes increases, however, the benefit of channel sensing becomes evident. CSMA_{TX} can then provide up to 18% less OP. The simple feedback channel introduced in CSMA_{RX} results in significant improvement in the performance of CSMA_{TX}, by approximately 30% for low densities. Finally, we

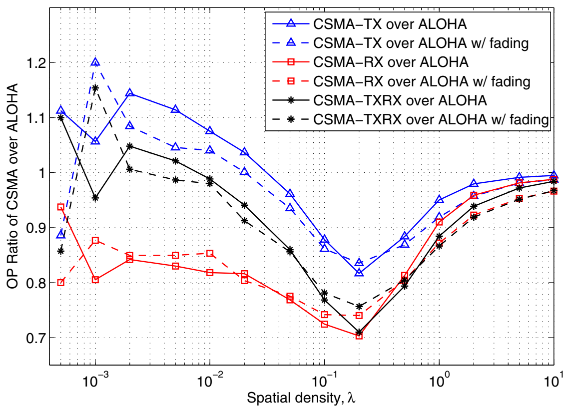

Fig. 2. Impact of fading on the OP of ALOHA and CSMA and their ratios, as a function of density λ.

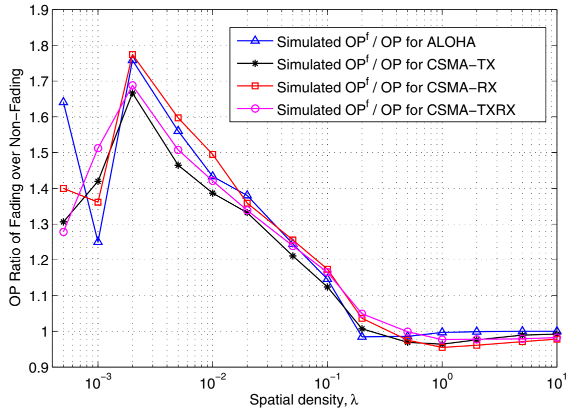

Fig. 3. Ratio of OP with fading over OP without fading, as a function of density λ.

observe that the joint decision making in $\mathrm{CSMA_{TXRX}}$ is not beneficial, as $\mathrm{CSMA_{RX}}$ outperforms it by up to 15%. Note that the OP of $\mathrm{CSMA_{RX}}$ is primarily due to the hidden node problem, while the OP of $\mathrm{CSMA_{TX}}$ and $\mathrm{CSMA_{TXRX}}$ is due to both the hidden (although smaller than in $\mathrm{CSMA_{RX}}$) and exposed node problems. As λ increases, the exposed node problem of $\mathrm{CSMA_{TX}}$ is reduced, while the hidden node problem increases, and the different flavors of CSMA behave more similarly.

Fig. 3 illustrates the impact of fading on the OP of ALOHA and CSMA. Compared to a non-faded network, the average OP in the presence of fading increases by up to 75% for ALOHA and $\mathrm{CSMA_{RX}}$ and up to 65% for $\mathrm{CSMA_{TX}}$ and $\mathrm{CSMA_{TXRX}}$. This is to be expected, as independent channel fading is known to degrade communication links. As the transmission density increases, the difference between the OP of a fading network and a non-fading network decreases. It is interesting to note that the change in the network performance due to fading is approximately the same for all the MAC protocols.

6. CONCLUSIONS AND FUTURE WORK

We have considered the performance of ALOHA and the various flavors of CSMA in a wireless ad hoc network with signal fading. With the introduction of fading it follows that we can no longer relate the outage probability to the distance between a RX and its interfering TXs. We rather consider the dominant interferers, which are active TXs whose individual interference powers are sufficient to cause an outage. Approximate expressions are derived for the OP of the different MAC protocols. It is established that compared to a non-faded network, the introduction of fading results in up to 65-75% higher OP for all the MAC protocols. As in non-fading networks, $\mathrm{CSMA_{RX}}$ yields the lowest OP, outperforming the conventional CSMA protocol by up to 30%. The inherent hidden and exposed node problems are the main contributing events to the OP of CSMA, where the latter is reduced by the simple feedback channel introduced in $\mathrm{CSMA_{RX}}$. Moreover, we observe that the relative change in the OP due to fading is approximately the same for all the MAC protocols.

For future work, we wish to add retransmissions to our network model, and analyze the transmission rate and delay of packets. Furthermore, the performance of the ad hoc network may be improved by means of power and rate adaptation.

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