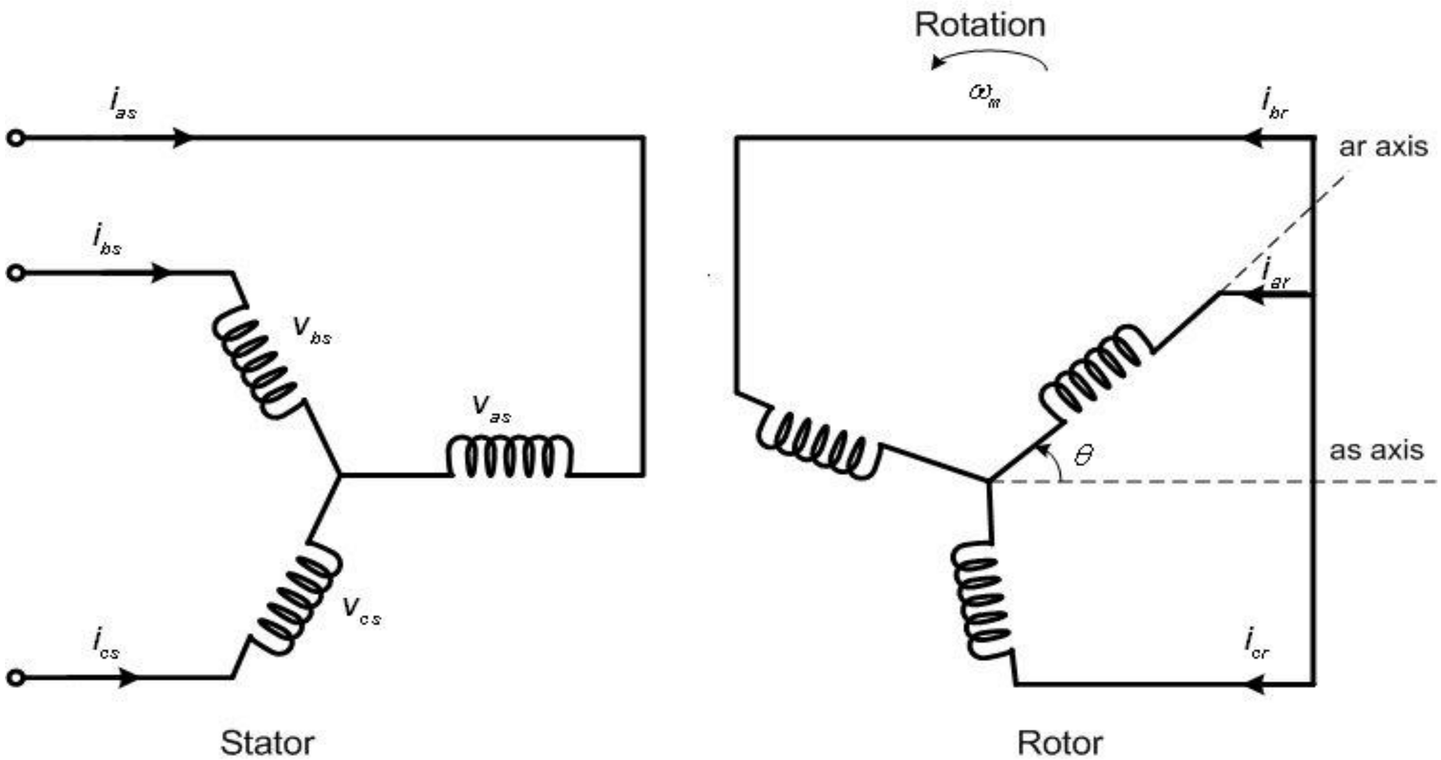
Matrix formulation of induction machine equations in phase variables:



Three-phase coupled circuit representation of an induction machine

Parameters:

$$R_{ss} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \qquad R_{rr} = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix}$$

$$L_{ss} = \begin{bmatrix} L_s & -.5L_m & -.5L_m \\ -.5L_m & L_s & -.5L_m \\ -.5L_m & -.5L_m & L_s \end{bmatrix} \qquad L_{rr} = \begin{bmatrix} L_r & -.5L_m & -.5L_m \\ -.5L_m & L_r & -.5L_m \\ -.5L_m & -.5L_m & L_r \end{bmatrix}$$

$$L_{sr} = L_m \begin{bmatrix} \cos(\theta) & \cos(\theta+\gamma) & \cos(\theta-\gamma) \\ \cos(\theta-\gamma) & \cos(\theta) & \cos(\theta+\gamma) \\ \cos(\theta+\gamma) & \cos(\theta-\gamma) & \cos(\theta) \end{bmatrix} \qquad L_{rs} = L_{sr}^t$$

$$\frac{dL_{sr}}{d\theta} = -L_m \begin{bmatrix} \sin(\theta) & \sin(\theta+\gamma) & \sin(\theta-\gamma) \\ \sin(\theta-\gamma) & \sin(\theta) & \sin(\theta+\gamma) \\ \sin(\theta+\gamma) & \sin(\theta-\gamma) & \sin(\theta) \end{bmatrix} \qquad \frac{dL_{rs}}{d\theta} = \frac{dL_{sr}^t}{d\theta}$$

$$\gamma = \frac{2\pi}{3}$$

Resistance matrix :

$$R = \begin{bmatrix} R_{ss} & 0 \\ 0 & R_{rr} \end{bmatrix}$$

Inductance matrix :

$$L = \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}$$

$$\frac{dL}{d\theta} = \begin{bmatrix} 0 & \frac{dL_{sr}}{d\theta} \\ \frac{dL_{rs}}{d\theta} & 0 \end{bmatrix}$$

Variables:

$$V = [v_{as}\, v_{bs}\, v_{cs}\, 0\; 0\; 0]^t$$

$$I = [i_{as}\, i_{bs}\, i_{cs}\, i_{ar} i_{br} i_{cr}]^t$$

where $v_{as} = V_s \cos(\omega_o t)$; $v_{bs} = V_s \cos(\omega_o t - \gamma)$; $v_{cs} = V_s \cos(\omega_o t + \gamma)$

Constitutive flux-current relations: $\lambda = LI$

Electrical system equations:

$$V = RI + \frac{d\lambda}{dt} = RI + \frac{d(LI)}{dt}$$

$$= \left(R + \omega_m \frac{dL}{d\theta}\right) I + L\frac{dI}{dt}$$

Electromagnetic torque:

$$T_e = \frac{\partial W_m}{\delta \theta_{mec}} = \frac{p}{2}\frac{1}{2}\frac{\partial}{\partial \theta}(I^t L I) = \frac{p}{4}\, I^t \frac{dL}{d\theta} I$$

where W_m = magnetic co-energy

Mechanical system equations:

$$T_e = J\frac{d\omega_{mec}}{dt} + B_m \omega_{mec} + T_L \quad ; \quad \theta = \frac{p}{2}\theta_{mec}$$

$$\omega_m = \frac{d\theta}{dt}$$

Nomenclature:

R_s = Stator resistance [Ohm]
R_r = Rotor resistance [Ohm]
L_{sl} = Stator leakage inductance [H]
L_{rl} = Rotor leakage inductance [H]
L_m = maximum single-phase mutual inductance between stator and rotor phase [H]
= $\frac{2}{3}$ magnetizing (3-phase) inductance
$L_{xy} = L_m cos(\theta_{xy})$
= Mutual inductance between a stator and a rotor phase

V_s = Peak phase voltage [V]
ω_o = Base frequency [rad/s]
J = Moment of inertia [kg.m^2]
B_m = Frictional coefficient
p = Number of poles
ω_m = Rotor speed [electrical rad/s]
θ = Rotor position [rad]